TRANSFER PROCESSES WITH ADDITION OF FUEL-OXIDIZER MIXTURE TO A REACTING BOUNDARY LAYER

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An investigation has been made of diffusion burning of a fuel-oxidizer mixture supplied through a porous plate to a compressible laminar boundary layer.

The present paper examines the question of laminar flow of air over a semi-infinite porous plate through which a fuel-oxidizer mixture is supplied according to the law $(\rho v)_w = \text{const}$. The mass flow rate of oxidizer is, in general, less than stoichiometric, and therefore burning cannot occur inside the plate. Insufficient oxygen reaches the burning zone from the external flow, and the reaction plane in the boundary layer corresponds to the point where there is a stoichiometric ratio, while the concentration of fuel and oxidizer tends to zero. It is assumed that the diffusion rate is negligibly small in comparison with the velocity of the chemical reaction occurring at a definite temperature T^* in an infinitely thin zone, which is a surface of discontinuity in the boundary layer.

It is assumed that $\Pr \neq 1 = \text{const}$, $\text{Sc} \neq 1 = \text{const}$ and $\Pr \neq \text{Sc}$, $c_p = \text{const}$, $\mu/\bar{\mu} = T/\bar{T}$, where $\bar{\mu}$ and $\bar{T}$ are constants [1, 2], $D = D(T)$, $\lambda = \lambda(T)$.

The system of equations of the laminar boundary layer in the case in question (without taking account of thermal diffusion) has the form

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial H}{\partial y}\right) + \frac{\partial}{\partial y} \times \left[\left(1 - \frac{1}{\Pr}\right)\mu \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right], \tag{3}$$

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{\text{Sc}_i}\frac{\partial C_i}{\partial y}\right], \tag{4}$$

$$C_i + C_f + C_o = 1. \tag{5}$$

The system of Eqs. (1)–(4) in the Crocco variables $u = u(x, y)$ and $x = x$, under the assumptions $\partial H/\partial x = 0$, $\partial C_i/\partial x = 0$, $dp/dx = 0$, and after excluding velocity v, transforms to the following form: the equation of conservation of energy

$$H'' - (\Pr - 1)\frac{K'}{K}H' = (1 - \Pr)\left[\eta \frac{K'}{K} + 1\right]u_\infty^2, \tag{6}$$

the diffusion equation

$$C_i'' + (1 - \text{Sc}_i)\frac{K'}{K}C_i' = 0, \tag{7}$$

the equation of conservation of momentum

$$KK'' + 2\eta\rho_0\mu_0 = 0, \tag{8}$$

where

$$\eta = u/u_\infty, \quad \rho_0 = \rho/\rho_\infty, \quad \mu_0 = \mu/\mu_\infty, \quad K = 2\tau\sqrt{x/\rho_\infty\mu_\infty u_\infty^3}$$

(derivatives with respect to η are denoted by primes). The quantity K depends on the blowing parameter.

In the simplest case, if $\rho_0\mu_0 = 1$, Eq. (8) is brought to a form independent of the general system

$$KK'' + 2\eta = 0. \tag{9}$$

Since (9) is equivalent to the Blasius equation, the function $K(\eta)$ may be determined from tables obtained in [1] for the Blasius problem.

The boundary conditions for the system (6)–(8) have the form

$$C_o = C_{o\infty},\ H = H_\infty,\ K = 0 \text{ when } \eta = 1, \tag{10}$$

$$C_i = 0, \quad H^I = H^{II} = H_*, \quad K(\eta)^I = K(\eta)^{II}, \tag{11}$$

$$-\frac{\tau_*}{\text{Sc}_m}\left(\frac{\partial C_f}{\partial \eta}\right)^{II} = a\left[\frac{\tau_*}{\text{Sc}_m}\left(\frac{\partial C_o}{\partial \eta}\right)^{I} - \frac{\tau_*}{\text{Sc}_m}\left(\frac{\partial C_o}{\partial \eta}\right)^{II}\right] \text{ when } \eta = \eta_*, \tag{11a}$$

$$K' = 2(\rho v)_w\sqrt{\frac{x}{\rho_\infty\mu_\infty u_\infty^3}},\ H = H_w,\ (\rho v)_w C_{ie} = (\rho v)_w C_{iw} - \frac{\tau_w}{u_\infty \text{Sc}_{\text{cm}}}C_{iw}' \text{ when } \eta = 0 \tag{12}$$

(here $i \equiv m$ and $i \equiv O_2$).

Condition (11a) follows from the equation of mass balance in the flame

$$-\left[\rho D_{fm}\frac{\partial C_f}{\partial y}\right]^{II} = a\left\{\left[\rho D_{om}\frac{\partial C_o}{\partial y}\right]^{I} - \left[\rho D_{om}\frac{\partial C_o}{\partial y}\right]^{II}\right\}.$$

Taking into account the equality $\rho_e v_e = (\rho v)_w$, we obtain the third condition in (12) from the mass balance of the injected component i

$$(\rho v)_w C_{ie} = (\rho v)_w C_{iw} - \left(\rho D_{im}\frac{\partial C_i}{\partial y}\right)_w. \tag{13}$$

To find the location of the surface with coordinate $\eta_* = \eta_*(x)$, a surface of discontinuity in the boundary layer, we must solve the diffusion Eq. (7) with boundary conditions (10), (11), and (11), (12), referring to regions I and II, respectively. From the solution we

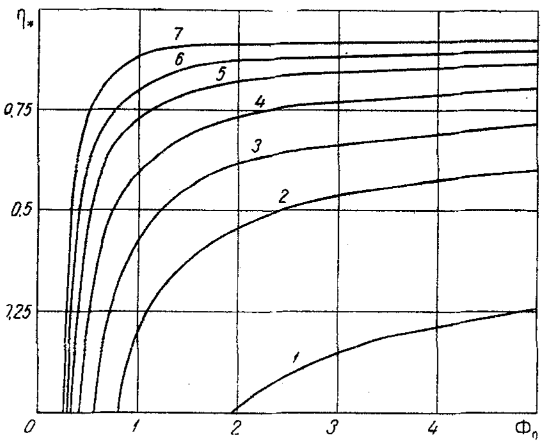

Fig. 1. Dependence of the position of the reaction front η_* on the parameter $\Phi_0 = C_{fe}/C_{Oe}$ (Sc = Pr = 1): 1) when B' = 0.1; 2) 0.2; 3) 0.3; 4) 0.5; 5) 1; 6) 2; 7) 10.

obtain relations determining the distribution concentration of oxygen (i ≡ O) and fuel (i ≡ f) in region II

$$C_i(\eta) = C_{ie} \int_{\eta_*}^{\eta} [K(\eta)/K(0)]^{Sc-1}\, d\eta \left\{ \int_{\eta_*}^{0} [K(\eta)/K(0)]^{Sc-1}\, d\eta - [u_\infty (\rho v)_w \, Sc/\tau_w]^{-1} \right\}^{-1} \quad (14)$$

and of oxygen in region I

$$C_O(\eta) = C_{O\infty} \int_{\eta_*}^{\eta} [K(\eta)/K(0)]^{Sc-1} \times d\eta \left\{ \int_{\eta_*}^{1} [K(\eta)/K(0)]^{Sc-1}\, d\eta \right\}^{-1}. \quad (15)$$

The distribution of concentrations C_I is determined by (5), since C_f and C_0 have been determined.

From (11a), taking account of (14) and (15), we obtain a relation giving the position η_* of the reaction front,

$$[K(\eta_*)/K(0)]^{\Pr m-1} (aC_{Oe} - C_{fe}) \int_{\eta_*}^{1} [K(\eta)/K(0)]^{Sc-1}\, d\eta = aC_{O\infty} \left\{ \int_{\eta_*}^{0} [K(\eta)/K(0)]^{Sc-1}\, d\eta - [u_\infty (\rho v)_w \, Sc/\tau_w]^{-1} \right\}, \quad (16)$$

where $0 < \eta \le 1$.

If we write

$$\tilde{I}(\eta_1, \eta_2) = \int_{\eta_1}^{\eta_2} [K(\eta_2)/K(\eta_1)]^{Sc-1}\, d\eta,$$

then (16) takes the form

$$(C_{fe} - aC_{Oe})[\tilde{I}(0,1) - \tilde{I}(0,\eta_*)] = aC_{O\infty}[\tilde{I}(0,\eta_*) + (B'\,Sc)^{-1}]. \quad (16a)$$

We solve the energy equation (6) by the method of variation of constants. Writing

$$J(\eta_1, \eta_2) = \int_{\eta_1}^{\eta_2} [K(\eta_2)/K(\eta_1)]^{\Pr-1} \int_{\eta_1}^{\eta_2} [K(\eta_2)/K(\eta_1)]^{1-\Pr}\, d\eta\, d\eta, \quad (17)$$

$$I(\eta_1, \eta_2) = \int_{\eta_1}^{\eta_2} [K(\eta_2)/K(\eta_1)]^{\Pr-1}\, d\eta, \quad (18)$$

for the enthalpy distribution in region II, we obtain

$$H - H_w = u_\infty^2 [\eta^2/2 - \Pr J(0,\eta)] + \{H_* - H_w - u_\infty^2 [\eta_*^2/2 - \Pr J(0,\eta_*)]\}\, I(0,\eta)/I(0,\eta_*) \quad (19)$$

(according to (17) and (18), in J(0, η) and I(0, η), $\eta_1 = 0$, $\eta_2 = \eta$). The expression for H in region I has the form

$$H - H_* = u_\infty^2 [(\eta^2/2 - \eta_*^2/2) - \Pr J(\eta_*, \eta)] + \{H_\infty - H_* - u_\infty^2 [(1/2 - \eta_*^2/2) - \Pr J(\eta_*, 1)]\}\, I(\eta_*, \eta)/I(\eta_*, 1). \quad (20)$$

Functions I and J have been tabulated [2] for various values of the Prandtl number and for the shear stress distribution obtained from solution of the Blasius problem. The functions $\tilde{I}(\eta, Sc)$ may be determined from tables for I(η, Pr). Thus, determining the position of the reaction front from (16), we may calculate the heat transfer between the porous surface and the stream

$$\alpha = \left(\lambda \frac{\partial H}{\partial y}\right)_w \Big/ (H_* - H_w) = \frac{\lambda \tau_w H'_w}{\mu u_\infty (H_* - H_w)}. \quad (21)$$

Since

$$\tau_w = \frac{1}{2} K(0) \sqrt{\rho_\infty \mu_\infty u_\infty^3 / x},$$

and the Reynolds number is

$$Re_x = \rho_\infty u_\infty x/\mu_\infty,$$

then

$$\tau_w = \frac{1}{2} K(0) \frac{u_\infty \mu_\infty}{x} \sqrt{Re_x}. \quad (21a)$$

Thus

$$\alpha = \frac{K(0)\sqrt{Re_x}}{2x(H_* - H_w)} \zeta(\eta_*),$$

where

$$\zeta(\eta_*) = \{H_* - H_w - u_\infty^2 [\eta_*^2/2 + \Pr J(0,\eta_*)]\}/I(0,\eta_*),$$

or

$$\frac{Nu}{\sqrt{Re_x}} = \frac{K(0)}{2(H_* - H_w)} \zeta(\eta_*). \quad (22)$$

The relations presented above are appreciably simplified under the condition Pr = Sc = 1. From (14)-(20) we obtain a linear dependence of concentration distribution and total enthalpy on velocity:

$$C_i(\eta) = C_{ie}(\eta_* - \eta)[\eta_* - 1/B']^{-1},$$

$$C_O(\eta) = C_{O\infty}(\eta - \eta_*)[1 - \eta_*]^{-1};$$

$$(H - H_w)/(H_* - H_w) = \eta/\eta_*,$$

$$(H - H_*)/(H_\infty - H_*) = (\eta - \eta_*)/(1 - \eta_*).$$

Equation (16) is transformed to the form

$$\eta_*(x) = [C_{fe} - a(C_{Oe} + C_{O\infty}/B')] \times$$

$$\times [C_{fe} - a(C_{Oe} - C_{O\infty})]^{-1}. \qquad (23)$$

For the calculation of heat transfer, we have, from (22),

$$\mathrm{Nu}/\sqrt{\mathrm{Re}_x} = 0.332/\eta_*. \qquad (24)$$

A relation similar to (24) was obtained in [3] in calculating a reacting boundary layer with discontinuity surface and injection of coolant gas according to the law $(\rho v)_w \sim x^{1/2}$.

As follows from analysis of relations (16) and (22)-(24), and also from Figs. 1 and 2 which are drawn for the case Pr = Sc = 1, $C_{O\infty} = 0.23$, $a = 0.25$, the heat transfer decreases as the intensity of blowing B' increases, while the reaction front moves further away from the plate surface.

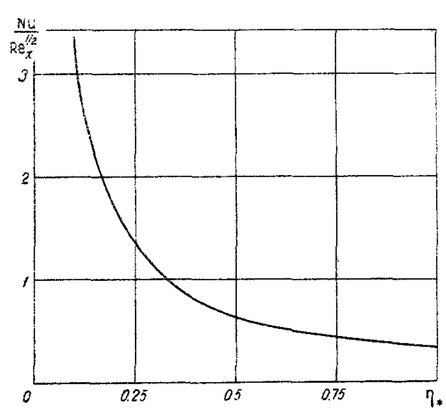

Fig. 2. Dependence of $\mathrm{Nu}/\mathrm{Re}_x^{1/2}$ on η_* (Pr = Sc = 1).

At small values of the parameter $\Phi_0 = C_{fe}/C_{Oe}$, $0 < \Phi_0 < 1$, which characterizes the mass content of fuel C_f and of oxygen C_0 in the injected mixture, a sharp change is observed in the position of the reaction front as Φ_0 increases (Fig. 1), while the maximum growth of η_* occurs at large values (B'). In this case $(0 < \Phi_0 < 1)$ the mixture arrives with a comparatively low fuel content. When the concentration C_f is increased, the flow of oxidizer

$$j_{O\infty} = -\rho_m D_{mO}\frac{\partial C_{O\infty}}{\partial y} \approx -\rho_m D_{mO}\frac{C_{O\infty} - C_O^*}{\delta - y^*}$$

(where δ is the boundary layer thickness) reaching the reaction front from the external stream must be increased, attaining the stoichiometric quantity at the burning zone near the body surface. In the first approximation we may consider that the sharp fall in gradient $\partial C_{O\infty}/\partial y$ corresponds to an insignificant change in C_f, and thus in the position of the reaction front (mass flux may not be taken into account without disturbing the qualitative picture of the process).

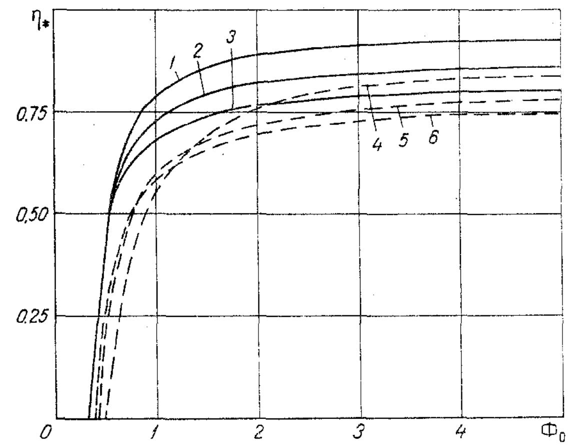

Fig. 3. Dependence of η_* on $\Phi_0 = C_{fe}/C_{Oe}$ for various values of Sc number and blowing parameter B': 1, 2, 3) with B' = 1.0 and Sc = 0.5, 1.0, and 1.5, respectively; 4, 5, 6) with B' = 0.5 and Sc = 0.5, 1.0, 1.5.

When $\Phi_0 > 1$ the value of the gradient $\partial C_{O\infty}/\partial y$ depends appreciably on the parameter Φ_0. The position η_* is determined mainly by B', η_* increasing with increase of B'.

As follows from the curves of $\eta_* = \eta_*(\Phi_0)$ shown in Fig. 3 and drawn in accordance with (16) and (23) for Sc numbers equal to 0.5, 0.1, and 1.5, and various values of B', the position of the reaction front η_* depends on many parameters with opposing influences (Φ_0, B', Sc). For a particular combination of these parameters the position of the reaction front η_* remains unchanged at various values of Sc.

Finally, let us examine the special case when a fuel-air mixture is injected into the boundary layer. Since the composition of the air may be considered homogeneous, the relation valid for the concentration of oxygen and fuel is

$$C_{Oe} = (1 - C_{fe})C_{O\infty}. \qquad (25)$$

Using the parameter

$$\Phi_f = C_{fe}(1 + aC_{O\infty})/aC_{O\infty}, \qquad (26)$$

which denotes the stoichiometric mass concentration of fuel in the fuel-air mixture [4], and taking (21a) and (25) into account, Eq. (16) may be transformed into

$$\Phi_f[\tilde I(0,1) - \tilde I(0,\eta_*)] - \tilde I(0,1) = 0.332(1\ \bar{\xi}\,\mathrm{Sc})^{-1}. \qquad (27)$$

where ξ is a dimensionless distance

$$\sqrt{\xi} = \frac{\rho_w v_w}{\rho_\infty u_\infty} \sqrt{\frac{\rho_\infty u_\infty x}{\mu_\infty}}.$$

When Sc = 1, we obtain from (27)

$$\eta_* = 1 - \frac{1}{\Phi_f}\left(1 + \frac{0.332}{\sqrt{\xi}}\right),$$

which agrees with [4].

When fuel gas alone is injected through the porous wall, condition (11a) may be written in the form

$$-\frac{1}{\mathrm{Sc_m}}\left(\frac{\partial C_f}{\partial \eta}\right)^{II} = \frac{a}{\mathrm{Sc_m}}\left(\frac{\partial C_o}{\partial \eta}\right)^{I},$$

and, in addition, at the discontinuity surface $\eta = \eta_*$ the relation

$$Q\frac{\mathrm{Pr_m}}{\mathrm{Sc_m}}\left(\frac{\partial C_f}{\partial \eta}\right) = h'_{II} - h'_I$$

is valid. This relation is used in [5] instead of (11a) in solving a problem similar to (6)–(12). Thus, the solution presented is more general than that of [5].

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